OCR
Oxford Cambridge and RSA

Monday 22 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 Marks)

1 A rocket is launched vertically upwards from rest. The initial mass of the rocket, including fuel and payload, is m_0 and the propulsion system ejects mass at a constant mass rate k with constant speed u relative to the rocket. The only other force acting on the rocket is its weight. The acceleration due to gravity is constant throughout the motion.

At time t after launch the speed of the rocket is v.

(i) Show that while mass is being ejected from the rocket $v = u\ln\left(\frac{m_0}{m_0 - kt}\right) - gt$. **[9]**

The rocket initially has 2400 kg of fuel which is ejected at a constant rate of $100\,\text{kg s}^{-1}$ with constant speed $3000\,\text{m s}^{-1}$ relative to the rocket.

(ii) Given that the rocket must reach a speed of $7910\,\text{m s}^{-1}$ before releasing its payload, find the maximum possible value of m_0. **[3]**

2 Fig. 2 shows a system in a vertical plane. A uniform rod AB of length $2a$ and mass m is freely hinged at A. The angle that AB makes with the horizontal is θ, where $-\frac{2}{3}\pi < \theta < \frac{2}{3}\pi$. Attached at B is a light spring BC of natural length a and stiffness $\frac{mg}{a}$. The other end of the spring is attached to a small light smooth ring C which can slide freely along a vertical rail. The rail is at a distance of a from A and the spring is always horizontal.

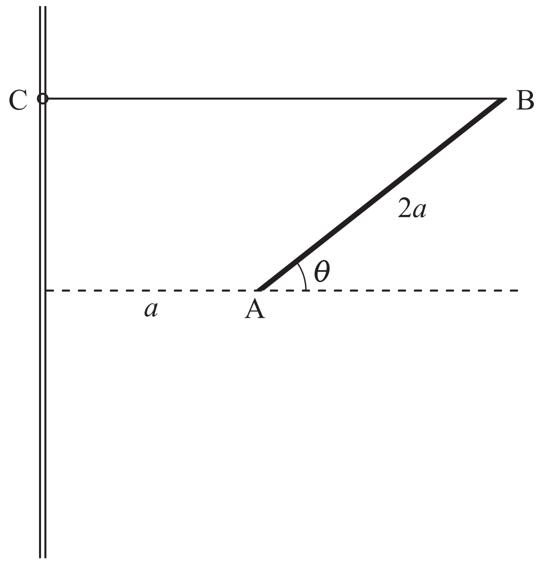

Fig. 2

(i) Find the potential energy, V, of the system and hence show that $\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga\cos\theta(1 - 4\sin\theta)$. **[5]**

(ii) Hence find the positions of equilibrium of the system and investigate their stability. **[7]**

Section B (48 Marks)

3 A particle of mass 4 kg moves along the x-axis. At time t seconds the particle is x m from the origin O and has velocity $v\,\mathrm{m\,s^{-1}}$. A driving force of magnitude $20te^{-t}$ N is applied in the positive x direction. Initially $v = 2$ and the particle is at O.

(i) Find, in either order, the impulse of the force over the first 3 seconds and the velocity of the particle after 3 seconds. **[8]**

From time $t = 3$ a resistive force of magnitude $\frac{1}{2}t$ N and the driving force are applied until the particle comes to rest.

(ii) Show that, after the resistive force is applied, the only time at which the resultant force on the particle is zero is when $t = \ln 40$. Hence find the maximum velocity of the particle during the motion. **[11]**

(iii) Given that the time T seconds at which the particle comes to rest is given by the equation $T = \sqrt{121 - 80e^{-T}(1+T)}$, without solving the equation deduce that $T \approx 11$. **[2]**

(iv) Use a numerical method to find T correct to 4 decimal places. **[3]**

4 A solid cylinder of radius a m and length $3a$ m has density $\rho\,\text{kg}\,\text{m}^{-3}$ given by $\rho = k\left(2 + \frac{x}{a}\right)$ where x m is the distance from one end and k is a positive constant. The mass of the cylinder is M kg where $M = \frac{21}{2}\pi a^3 k$. Let A and B denote the circular faces of the cylinder where $x = 0$ and $x = 3a$, respectively.

(i) Show by integration that the moment of inertia, $I_A\,\text{kg}\,\text{m}^2$, of the cylinder about a diameter of the face A is given by $I_A = \frac{109}{28}Ma^2$. **[9]**

(ii) Show that the centre of mass of the cylinder is $\frac{12}{7}a$ m from A. **[4]**

(iii) Using the parallel axes theorem, or otherwise, show that the moment of inertia, $I_B\,\text{kg}\,\text{m}^2$, of the cylinder about a diameter of the face B is given by $I_B = \frac{73}{28}Ma^2$. **[4]**

You are now given that $M = 4$ and $a = 0.7$. The cylinder is at rest and can rotate freely about a horizontal axis which is a diameter of the face B as shown in Fig. 4. It is struck at the bottom of the curved surface by a small object of mass 0.2 kg which is travelling horizontally at speed $20\,\text{m}\,\text{s}^{-1}$ in the vertical plane which is both perpendicular to the axis of rotation and contains the axis of symmetry of the cylinder. The object sticks to the cylinder at the point of impact.

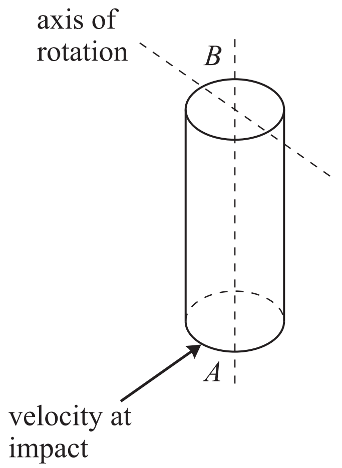

Fig. 4

(iv) Find the initial angular speed of the combined object after the collision. **[7]**

END OF QUESTION PAPER

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4764/01 Mechanics 4

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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Turn over

Section A (24 marks)

1 (i)	
	(answer space continued on next page)

1 (i)	**(continued)**
1 (ii)	

2 (i)	

2 (ii)	

Section B (48 marks)

3 (i)

3 (ii)	

3 (iii)	
3 (iv)	

4 (i)

4 (ii)	
4 (iii)	
	(answer space continued on next page)

4 (iii)	**(continued)**
4 (iv)	
	(answer space continued on next page)

4 (iv)	(continued)

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance
1	(i)		$(m+\delta m)(v+\delta v)-\delta m(v-u)-mv=-mg\delta t$	M1*	Attempt at momentum equation; 4 terms (allow one slip or sign errors)
			NB $\delta m < 0$	A1	Condone wrong sign of δm
			$m\dfrac{\delta v}{\delta t}+u\dfrac{\delta m}{\delta t}+\delta m\dfrac{\delta v}{\delta t}=-mg$	M1 dep*	Simplify and divide by δt
			$m\dfrac{\mathrm{d}v}{\mathrm{d}t}+u\dfrac{\mathrm{d}m}{\mathrm{d}t}=-mg$	A1	Complete argument including sign of δm correct – if differential equation stated rather than derived then SC B3
			$m = m_0 - kt$	B1	Seen or implied
			$\dfrac{\mathrm{d}v}{\mathrm{d}t}=\dfrac{uk}{m_0-kt}-g$	M1	Substituting their $m_0 - kt$ into their three term differential equation which contains two derivatives
			$v=\int\left(\dfrac{uk}{m_0-kt}-g\right)\mathrm{d}t$	M1	Separation of variables - must be of the correct form $v=\int\left(\dfrac{A}{B-Dt}-g\right)\mathrm{d}t$ and attempt to integrate
			$=-u\ln(m_0-kt)-gt(+c)$	A1	
			When $t = 0$, $v = 0 \Rightarrow c = u\ln m_0$		
			$v=u\ln\left(\dfrac{m_0}{m_0-kt}\right)-gt$	E1	
				[9]	
	(ii)		$t = 24$ sec	B1	
			$7910=3000\ln\left(\dfrac{m_0}{m_0-2400}\right)-9.8\times 24$	M1	Substitute into given equation and attempt to solve
			m_0 =2570.14…. so 2570 kg (3sf)	A1	
				[3]	

2	**(i)**		$V = mga\sin\theta$ with datum level through A	B1	
			$+\ \frac{1}{2}\frac{mg}{a}(2a\cos\theta)^2$	M1* A1	Genuine attempt at extension + substitution into $\frac{1}{2}kx^2$ $V = mga\left(\sin\theta + 1 + \cos 2\theta\right) = mga\left(\sin\theta + 2\cos^2\theta\right)$
			$\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga\cos\theta - 4mga\cos\theta\sin\theta$	M1 dep*	Differentiates their V of the correct form
			$= mga\cos\theta(1 - 4\sin\theta)$	E1 **[5]**	
	(ii)		$mga\cos\theta(1-4\sin\theta) = 0$	M1	
			$\cos\theta = 0 \Rightarrow \theta = \frac{1}{2}\pi\ ,-\frac{1}{2}\pi$	A1	
			$\sin\theta = \frac{1}{4} \Rightarrow \theta = \sin^{-1}\frac{1}{4}$	A1	For this root and rejection of $\pi - \arcsin(0.25)$ (accept either no reason given for rejection or sin θ = 1/4 (only) and the interval for θ stated)
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -mga\sin\theta - 4mga\cos 2\theta$	M1	May use $\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga\left(\cos\theta - 2\sin 2\theta\right)$ giving $-mga\left(\sin\theta + 4\cos 2\theta\right)$ or $-mga\left(\sin\theta + 4 - 8\sin^2\theta\right)$
			When $\theta = \frac{1}{2}\pi,\ \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -mga + 4mga > 0$ so stable	A1	For all A marks accept (as a minimum) correct V'' with corresponding sign of V'' + correct conclusion (if values of V'' seen then they must be correct)
			When $\theta = -\frac{1}{2}\pi,\ \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga + 4mga > 0$ so stable	A1	
			When $\theta = \sin^{-1}\frac{1}{4},\ \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -\frac{1}{4}mga - \frac{7}{2}mga < 0$ so unstable	A1 **[7]**	

3	(i)		$J = \int_a^b f(t)\mathrm{d}t = \int_0^3 20te^{-t}\mathrm{d}t$	B1	OR $\int_2^v 4dv = \int_0^3 20te^{-t}\mathrm{d}t$
			$= \left[-20te^{-t}\right]_0^3 + \int_0^3 20e^{-t}\mathrm{d}t$	M1* A1	Use integration by parts (no limits required for M1 A1 A1) Correctly applied to end of first stage
			$= \left[-20te^{-t} - 20e^{-t}\right]_0^3$	A1	
			$= -3.98296\ldots + 20$ $= 16.01703\ldots$ $= 16.0$ Ns	M1 dep* A1	$20 - 80e^{-3}$ Award only when identified as J
			$J = \Delta p$ $16.01703\ldots = 4v - 8$ $v = 6.00425\ldots$ $= 6.00\ \mathrm{ms}^{-1}$	M1 A1ft	Working may be seen above $7 - 20e^{-3}$
				[8]	
			Alternative find v first: $4\frac{\mathrm{d}v}{\mathrm{d}t} = 20te^{-t}$	B1	
			$v = -5te^{-t} + \int 5e^{-t}\mathrm{d}t$	M1* A1	M1 Use integration by parts, A1 correctly applied to end of first stage
			$v = -5te^{-t} - 5e^{-t}(+c)$	A1	
			$t = 0, v = 2 \Rightarrow c = 7$	M1 dep*	Use initial conditions
			$v = 6.00425\ldots$	A1	
			$J = 4(6.00425\ldots - 2) = 16.017\ldots$	M1 A1ft	

	(ii)		$F = 20te^{-t} - \frac{1}{2}t = 0$	B1	
			$t(20e^{-t} - \frac{1}{2}) = 0$, so $20e^{-t} - \frac{1}{2} = 0$		
			$t = \ln 40$	E1	
			$t = 0$ not applicable (since $t \geq 3$ so this is the only stationary point)	E1	Stating that $t = 0$ is not applicable (must clearly state that $t \neq 0$ but no reason necessary, this mark can be awarded if explicitly stating that $t \geq 3$)
			F and hence a change sign (from positive to negative) either side of $t = \ln 40$	B1	Any correct argument that v is a maximum when $t = \ln 40$ (maybe seen later)
			$4\frac{dv}{dt} = 20te^{-t} - \frac{1}{2}t$	B1	
			$\int 4dv = \int \left(20te^{-t} - \frac{1}{2}t\right)dt$	M1*	Separate and attempt to integrate; using v from part (i) is M0
			$4v = -20e^{-t}(1+t) - \frac{1}{4}t^2(+c)$	A2	A1 for one error
			When $t = 3$, $v = 6.00425\ldots$	M1 dep*	Use initial conditions
			$\Rightarrow c = 30\frac{1}{4}$		($c = 7\frac{9}{16}$ if $v = \ldots$ form used)
			When $t = \ln 40$, $v = 6.1259\ldots$	M1	Dependent on both previous M marks – substitute $t = \ln 40$ and solving for v
			$v = 6.13 \text{ ms}^{-1}$	A1 **[11]**	Cao
	(iii)		$T = \sqrt{121 - 80e^{-T}(1+T)}$	B1	$80e^{-11}(1+11)$ as being small (oe)
			Justify T is approximately 11	E1 **[2]**	Considers e^{-T} becoming small
	(iv)		Eg a starting value of 11		
			10.99927117…	M1	A suitable method
			10.99927069…	M1	Correct working seen – must obtain an answer to at least 4dp
			so $t = 10.9993$ to 4 dp	A1 **[3]**	Justifiable conclusion from their working (10.9993 only – must be 4dp)

4	(i)		Mass of an elemental disc $\delta m = k\left(2+\frac{x}{a}\right).\pi a^2\,\delta x$	B1	Or $\delta m = \rho\pi a^2\delta x$ - condone lack of δx
			About diameter of elemental disc $\delta I = \frac{1}{4}\left(k\left(2+\frac{x}{a}\right).\pi a^2\,\delta x\right).a^2$	B1	Or $\delta I = \frac{1}{4}\left(\rho\pi a^2\delta x\right)a^2$ - condone lack of δx
			By the parallel axes theorem, about the given axis $\delta I = \frac{1}{4}\left(k\left(2+\frac{x}{a}\right).\pi a^2\,\delta x\right).a^2 + k\left(2+\frac{x}{a}\right).\pi a^2\,\delta x.x^2$	M1*	
			$= k\left(2+\frac{x}{a}\right).\left(\frac{1}{4}a^2 + x^2\right).\pi a^2\,\delta x$	A1	Must be of the form $\rho\left(\lambda a^2 + \mu x^2\right)a^2$ - condone lack of δx
			$I =$ $k\pi a^2\int_0^{3a}\left(2+\frac{x}{a}\right)\left(\frac{1}{4}a^2+x^2\right)\mathrm{d}x = k\pi a^2\int_0^{3a}\frac{1}{2}a^2+\frac{1}{4}ax+2x^2+\frac{1}{a}x^3\mathrm{d}x$	M1dep* A1	Condone lack of limits for both M1 A1 A1 correct simplified integral
			$= k\pi a^2\left[\frac{1}{2}a^2x+\frac{1}{8}ax^2+\frac{2}{3}x^3+\frac{1}{4a}x^4\right]_0^{3a}$	M1	Integrating and using correct limits - dependent on both previous M marks
			$= \frac{327}{8}k\pi a^5$	A1	
			$= \frac{327}{8}\frac{2M}{21\pi a^3}\pi a^5 = \frac{109}{28}Ma^2$	E1 **[9]**	If δx omitted throughout then withhold final mark
	(ii)		$M\bar{x} = \int x\,\mathrm{d}m = \int_0^{3a} k\left(2+\frac{x}{a}\right)\pi a^2 x\,\mathrm{d}x$	B1	Condone lack of limits
			$= k\pi a^2\int_0^{3a}2x+\frac{1}{a}x^2\mathrm{d}x = k\pi a^2\left[x^2+\frac{1}{3a}x^3\right]_0^{3a}$	M1	Integrating and using correct limits
			$= 18k\pi a^4$	A1	
			$= 18.\frac{2M}{21\pi a^3}.\pi a^4 = \frac{12}{7}Ma \Rightarrow \bar{x} = \frac{12}{7}a$	E1 **[4]**	

4	(iii)		$I_B = I_G + M d_{BG}^2$ $I_A = I_G + M d_{AG}^2$	M1	Attempt at use of parallel axis theorem (for both) (Note: $d_{BG} = \frac{9}{7}a, d_{AG} = \frac{12}{7}a, I_A = \frac{109}{28}Ma^2$)
			$I_B = (I_A - M d_{AG}^2) + M d_{BG}^2$	A1	Eliminate or evaluate I_G ($I_G = \frac{187}{196}Ma^2$)
			$= \frac{109}{28}Ma^2 + M\left(\frac{81}{49} - \frac{144}{49}\right)a^2$	A1	
			$= \frac{73}{28}Ma^2$	E1	
				[4]	
	(iv)		Moment of momentum of object about axis before collision is	M1	$0.2\times20\times(2.1 \text{ or } 7/\sqrt{10})$
			$0.2\times20\times2.1 = 8.4$	A1	M1 A1 for 8.4 seen (www)
			MI of object about axis after coalesces is $0.2\left(\frac{7}{\sqrt{10}}\right)^2 = 0.98$	B1	
			MI of cylinder about the axis is 5.11	B1	
			MI of combined object about axis is 6.09	B1	5.992 with no working scores B1 only
			Conservation of angular momentum $8.4 = 6.09\dot{\theta}$	M1	Use of conservation of angular momentum
			$\dot{\theta} = 1.379\ldots$	A1	Cao
				[7]	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

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GCE Mathematics (MEI)											
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	48	43	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	50	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	56	51	46	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	74	67	60	54	48	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	62	57	53	49	45	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	38	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	62	54	46	39	32	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	54	47	40	33	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	65	60	55	50	46	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	64	58	52	47	42	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	54	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	45	40	34	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	56	51	46	41	37	0
		UMS	100	80	70	60	50	40	0